FUNCTION PROJECTIVE SYNCHRONIZATION OF A NEW HYPER CHAOTIC SYSTEM

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Abstract- In this article a function projective synchronization (FPS) of two identical new hyper chaotic systems is defined and scheme of FPS is developed by using Open-Plus-Closed-Looping (OPCL) coupling method. A new hyper chaotic system has been constructed and then response system with parameters perturbation and without perturbation. Numerical simulations verify the effectiveness of this scheme, which has been performed by mathematica.

Index Term: Function Projective Synchronization, Chaotic systems and Hyper Chaos, OPCL.

1. **Introduction**

Chaos is a dynamical regime in which a system becomes extremely sensitive to initial conditions and reveals an unpredictable and random-like behavior, even though the underlying model of a system exhibiting chaos can be deterministic and very simple. Small differences in initial conditions yield widely diverging outcomes for chaotic systems, rendering long term prediction impossible in general.

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Chaotic behavior can be observed in many natural phenomenon such as weather etc. Pecora and Carroll introduced a paper entitled *Synchronization in Chaotic Systems* in 1990. By that time, if there was a system challenging the capability of synchronizing that was a chaotic one. They demonstrated that chaotic synchronization could be achieved by driving or replacing one of the variables of a chaotic system with a variable of another similar chaotic device. Chaotic synchronization did not attract much attention until Pecora and Carroll [4] introduced a method to synchronize two identical chaotic systems with different initial conditions. From then on, enormous studies have been done by researchers on the synchronization of dynamical systems[1, 2, 3]. In the last two decades considerable research has been done in non-linear dynamical systems and their various properties. One of the most important properties is synchronization.

Synchronization techniques have been improved in recent years and many different methods are applied theoretically as well as experimentally to synchronize the chaotic-systems including adaptive control [5, 6, 7], backstepping design [8, 9, 10], active control [11, 12, 13], nonlinear control [14, 15] and observer based control method [16]. Using these methods, numerous synchronization problem of well-known chaotic systems such as Lorenz, Chen, L¨u and R¨ossler system have been worked on by many researchers.

Also, several types of chaos synchronization are well known, which include complete synchronization (CS), antisynchronization (AS), phase synchronization, generalized synchronization (GS), projective synchronization(PS), and modified projective synchronization (MPS). Among all type of synchronizations, projective synchronization (PS) [17, 20, 21, 22] has been extensively considered because it can obtain faster communication. The drive and response system could be synchronized up to a scaling factor in projective synchronization. In this continuation of study, in order to increase the degree of secrecy for secure communications, function projective synchronization (FPS) [23] is characterized by a scaling function matrix. In this paper, we have constructed a new hyper chaotic system and verified the chaotic behavior of this system by time series analysis and chaotic attractors via mathematica. Hyperchaotic behavior of this system is discovered within some system parameters range, which has not yet been reported previously.

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Since hyperchaotic systems have the characteristics of high capacity, high security and high efficiency, it has been studied with increasing interest in recent years [19, 20] in the fields of non-linear circuits, secure communications, lasers, control, synchronization, and so on. So, we have studied Function Projective Synchronization behavior for this new hyper chaotic systems, which is ofcourse more effective and useful in secure communication as FPS is more useful in secure communication as compare to others because of its unpredictability . Here we have used OPCL coupling scheme for FPS. Numerical simulations have been done by using Mathematica.

2. Preliminaries

In this section we mention some definitions and scheme of the main task.

2.1. Function Projective Synchronization. Function Projective synchronization is defined in the following manner:

Let $\dot{x} = F(x, t)$ be the drive chaotic system, and $\dot{y} = F(y, t)+U$ is the response system, where $x = (x_1(t), x_2(t),\dots x_m(t))^T$, $y = (y_1(t), y_2(t), \dots y_m(t))^T$, $U = (u_1(x, y), u_2(x, y),\dots u_m(x, y))^T$ is a controller to be determined later.

Denote $e_i = x_i - f_i(x)y_i$; $(i = 1, 2,\dots m)$, $f_i(x)$; $(i = 1; 2;\dots;m)$ are functions of x. If

$$\lim_{t\to\infty}\|e(t)\|=0,$$

$e = (e_1; e_2; :::; e_m)$, then there exists function projective synchronization (FPS) between these two identical chaotic (hyperchaotic) systems, and we call f a scaling function matrix. Here we use the OPCL coupling method for FPS.

2.2. Methodology for FPS via OPCL. Here, we will construct corresponding response system through the OPCL coupling method. Consider the following n-dimensional chaotic system as drive (master) system

$$\frac{dx}{dt} = f(x) + \Delta f(x) \qquad (1)$$

where $x \in \Re^n$ and $\Delta f(x)$ is the perturbation part of the parameters. Now, consider the following n-dimensional chaotic system as responsive system according to coupling method

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$$\frac{dy}{dt} = f(y) + D(y,g), \qquad (2)$$

where $y \in \Re^n$. The coupling function is:

$$D(y,g) = \dot{g} - f(g) + \left(H - \frac{\partial f(g)}{\partial g}\right) .(y-g),$$

where $\frac{\partial f(g)}{\partial g}$ is the jacobian matrix of the dynamical system. H is an $n \times n$ Hurwitz constant matrix, whose eigen values are negative and $g = \beta(t)x$ with $\beta(t)$ as a scaling function which is continuously differentiable. When $\beta(t) = \pm 1$, system is complete synchronized or antisynchronized accordingly.

Our goal in this paper is to find out $D(y, g)$ and hence find error dynamics of the system such that

$$\lim_{t \to \infty} \|e(t)\| = \|y - g\| = 0$$

where $\|.\|$ is the Euclidean norm, then the systems (1) and (2) are said to be Function Projective synchronized.

3. System Description

3.1. Hyper Chaotic Rabinovich-Fabrikant system.

The Rabinovich-Fabrikant chaotic system is a set of three coupled ordinary differential equations exhibiting chaotic behavior for certain values of parameters.

They are named after Mikhail Rabinovich and Anatoly Fabrikant, who described them in 1979 [18]. The equations of system are :

$$\begin{cases} \dot{x}_1 = x_2(x_3 - 1 + x_1^2) + \gamma x_1, \\ \dot{x}_2 = x_1(3x_3 + 1 - x_1^2) + \gamma x_2, \\ \dot{x}_3 = -2x_3(x_1 x_2 + \alpha). \end{cases}$$

where α *and* γ are constant parameters that control the evolution of the system. For some values of α *and* γ the system is chaotic but for other it tends to a stable periodic orbit.

Now, we construct a new hyper chaotic system by introducing one more differential equation with a new parameter δ in the above system as follows:

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$$\begin{cases} \dot{x}_1 = x_2(x_3 - 1 + x_1^2) + \gamma x_1, \\ \dot{x}_2 = x_1(3x_3 + 1 - x_1^2) + \gamma x_2, \\ \dot{x}_3 = -2x_3(x_1 x_2 + \alpha), \\ \dot{x}_4 = -3x_3(x_2 x_4 + \delta) + x_4^2. \end{cases}$$
(4)

This new system shows hyper chaotic behavior with some values of parameters and tend to stable periodic orbits with other values of parameters. We have investigated system's behavior for different values of parameters

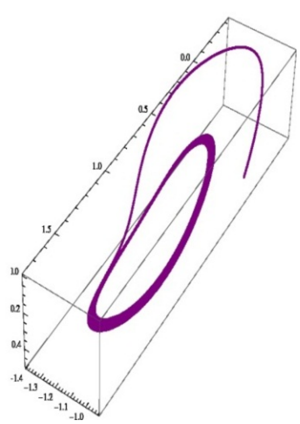

Fig.1
Chaotic behavior of the system with $\alpha = 0.14, \gamma = 1{:}1$ and $-0.01 \leq \delta \leq 7650$ tending to stable periodic orbits.

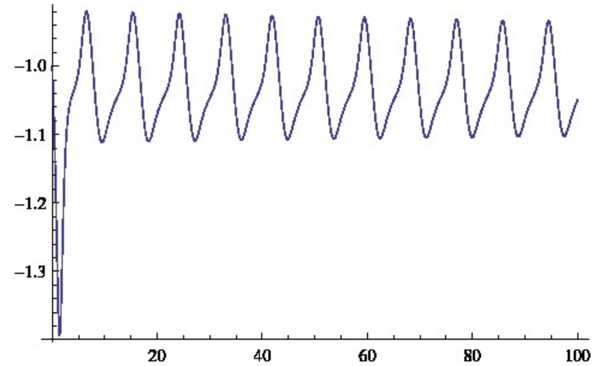

Fig.2 Time series analysis of $y_1[t]$ with $\alpha = 0.14, \gamma = 1{:}1$ and $-0.01 \leq \delta \leq 7650$.

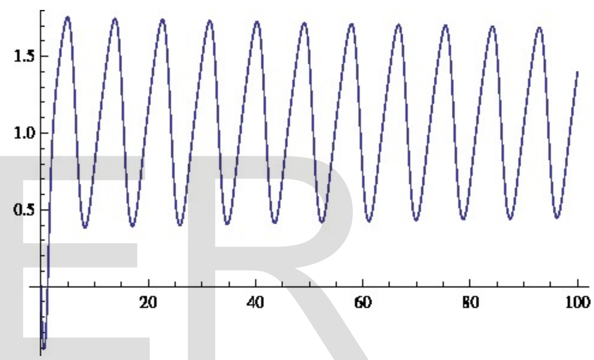

Fig.3 Time series analysis of $y_2[t]$ with $\alpha = 0.14, \gamma = 1{:}1$ and $-0.01 \leq \delta \leq 7650$.

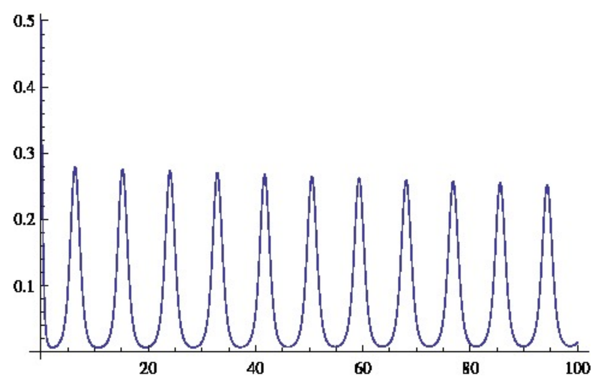

Fig.4 Time series analysis of $y_3[t]$ with $\alpha = 0.14, \gamma = 1{:}1$ and $-0.01 \leq \delta \leq 7650$

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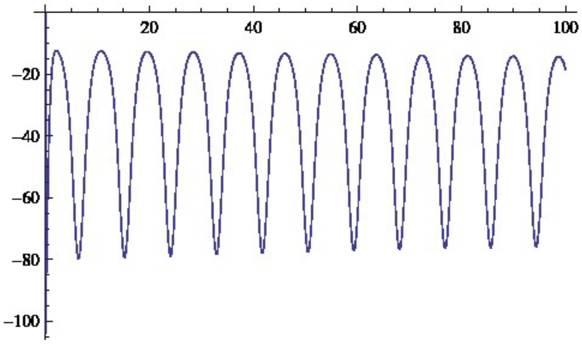

Fig.5 Time series analysis of y_4[t] with α = 0.14,γ = 1:1 and −0.01 ≤ δ ≤ 7650.

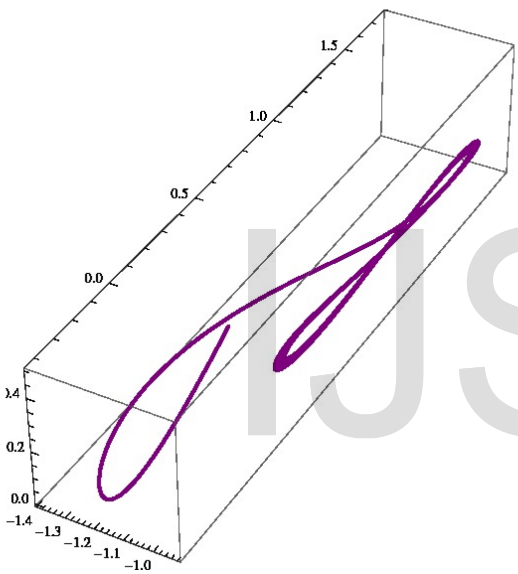

Fig.6 Chaotic Behavior of the system with α = 0:87, γ = 1:1 and δ=1890.

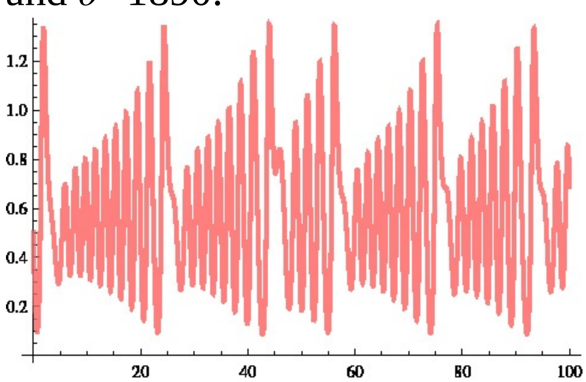

Fig.7 Time series analysis of y_1 [t] with α = 0:87, γ = 1:1 and δ=1890.

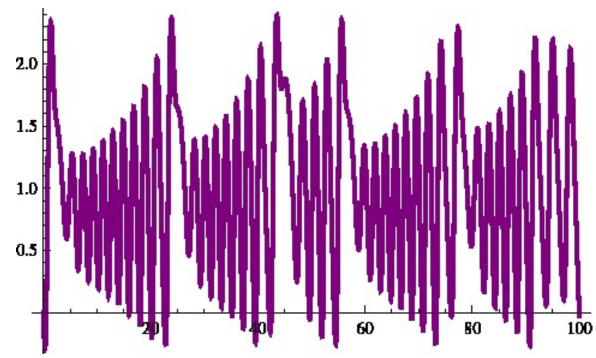

Fig.8 Time series analysis of y_2[t] with α = 0:87, γ = 1:1 and δ=1890.

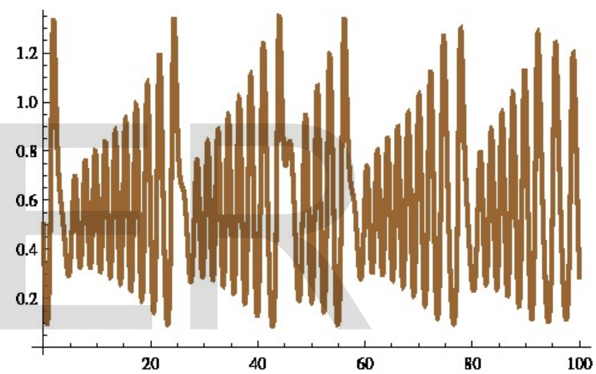

Fig.9 Time series analysis of y_3[t] with α = 0:87, γ = 1:1 and δ=1890.

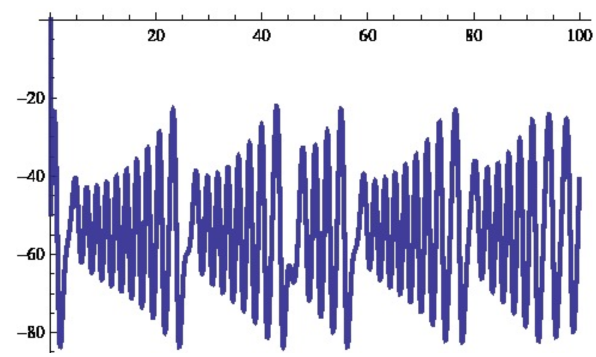

Fig.10 Time series analysis of y_4[t] with α = 0:87, γ = 1:1 and δ=1890.

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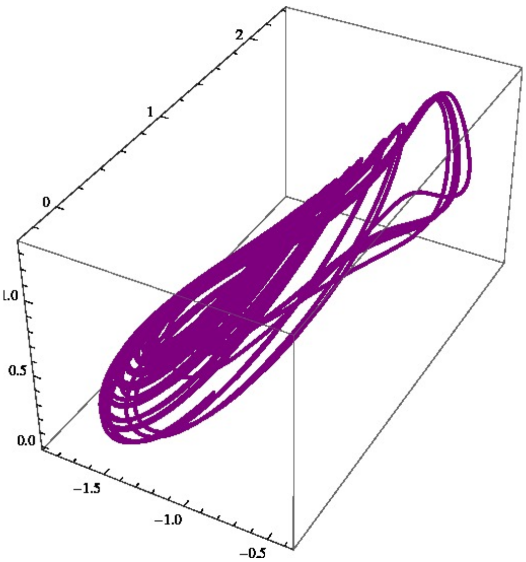

Fig.11 Chaotic Behavior of the system with $\alpha = 0{:}87$, $\gamma = 1{:}1$ and δ=-0.2.

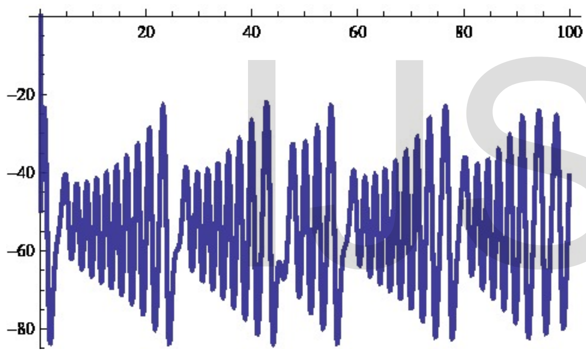

Fig.12 Time series analysis of y_1[t] with $\alpha = 0{:}87$, $\gamma = 1{:}1$ and δ=-0.2.

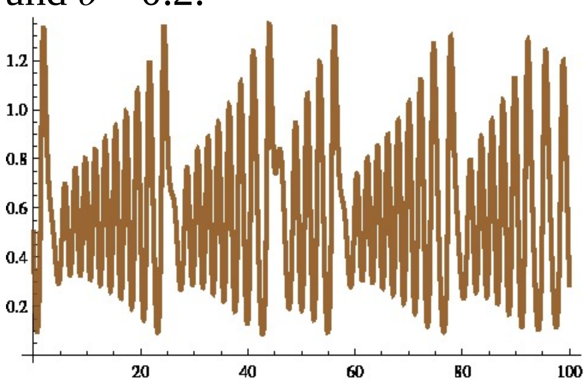

Fig.13 Time series analysis of y_2[t] with $\alpha = 0{:}87$, $\gamma = 1{:}1$ and δ=-0.2.

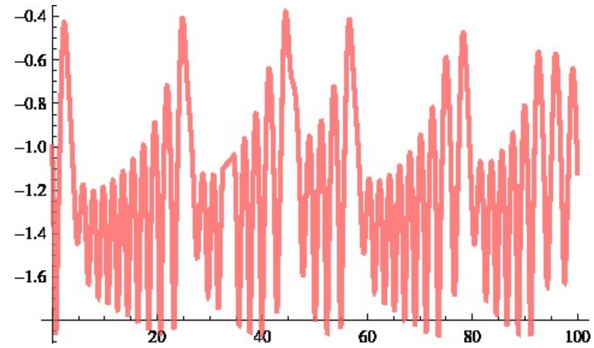

Fig.14 Time series analysis of y_3[t] with $\alpha = 0{:}87$, $\gamma = 1{:}1$ and δ=-0.2.

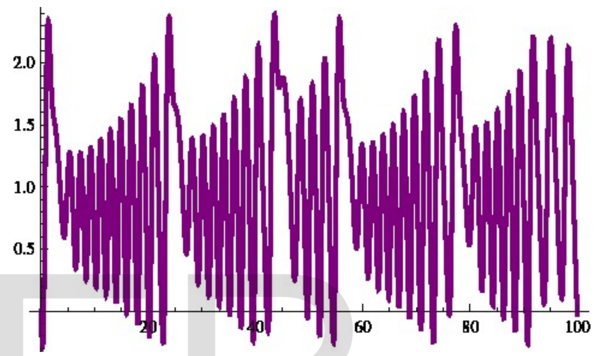

Fig.15 Time series analysis of y_4[t] with $\alpha = 0{:}87$, $\gamma = 1{:}1$ and δ=-0.2.

3.2. Results and Discussions.

In this section, we perform function projective synchronization of above described system via OPCL coupling method. Define following system as a drive system with parameters perturbation as

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$$\dot{x}_1 = x_2(x_3 - 1 + x_1^2) + (\gamma + \Delta\gamma)x_1,$$

$$\dot{x}_2 = x_1(3x_3 + 1 - x_1^2) + (\gamma + \Delta\gamma)x_2,$$

$$\dot{x}_3 = -2x_3(x_1x_2 + \alpha + \Delta\alpha),$$

$$\dot{x}_4 = -3x_3(x_2x_4 + \delta + \Delta\delta) + x_4^2.$$

where $\Delta\alpha$, $\Delta\gamma$ and $\Delta\delta$ are the perturbation parts in the parameters. Now construct the corresponding response system via OPCL coupling method.

The Jacobian matrix of the above system is

$$\frac{\partial f(x)}{\partial x} = \begin{pmatrix} \gamma + \Delta\gamma + 2x_1x_2 & x_1^2 + x_3 - 1 & x_2 & 0 \\ 3x_3 - 3x_1^2 + 1 & \gamma + \Delta\gamma & 3x_1 & 0 \\ -2x_2x_3 & -2x_1x_3 & -2\alpha - 2\Delta\alpha - 2x_1x_2 & 0 \\ 0 & -3x_3x_4 & -3x_2x_4 - 3\delta - 3\Delta\delta & 2x_4 - 3x_2x_3 \end{pmatrix}$$

Define Hurwitz matrix H as the unit negative matrix $-I$ (as $g = \beta(t)x$),

then $H - \frac{\partial f(g)}{\partial g}$ =

$$= \begin{pmatrix} -\gamma - \Delta\gamma + 2\beta^2 x_1x_2 - 1 & -\beta^2 x_1^2 - \beta x_3 + 1 & -\beta x_2 & 0 \\ -3\beta x_3 + 3\beta^2 x_1^2 - 1 & -\gamma - \Delta\gamma - 1 & -3\beta x_1 & 0 \\ 2\beta^2 x_2x_3 & 2\beta^2 x_1x_3 & 2\alpha + 2\Delta\alpha + 2\beta^2 x_1x_2 - 1 & 0 \\ 0 & 3\beta^2 x_3x_4 & 3\beta^2 x_2x_4 + 3\delta + 3\Delta\delta & -2\beta x_4 + 3\beta^2 x_2x_3 - 1 \end{pmatrix}$$

Therefore, response system after coupling is as follows

$$\dot{y}_1 = f(y_1) - f(g_1) + \dot{g}_1 + (H - \frac{\partial f(g_1)}{\partial g_1})(y_1 - g_1),$$

$$\dot{y}_2 = f(y_2) - f(g_2) + \dot{g}_2 + (H - \frac{\partial f(g_2)}{\partial g_2})(y_2 - g_2),$$

$$\dot{y}_3 = f(y_3) - f(g_3) + \dot{g}_3 + (H - \frac{\partial f(g_3)}{\partial g_3})(y_3 - g_3),$$

$$\dot{y}_4 = f(y_4) - f(g_4) + \dot{g}_4 + (H - \frac{\partial f(g_4)}{\partial g_4})(y_4 - g_4). \quad (6)$$

As error dynamics is defined as $e^{\cdot} = y^{\cdot} - g^{\cdot}$, so we have final equation of error dynamics after coupling and putting values of f(y), f(g) and $H - \frac{\partial f(g)}{\partial g}$ in above equation as follows

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$$
\begin{aligned}
\dot{e}_1 &= \Delta\gamma e_1 + e_2 e_3 + e_2 e_1^2 + 2\beta x_1 e_1 e_2 \\
&\quad + \beta x_1 e_1^2 - e_1, \\
\dot{e}_2 &= \Delta\gamma e_2 + 3e_1 e_3 + e_1^3 + 3\beta x_1 e_1^2 - e_2, \\
\dot{e}_3 &= -2\Delta\alpha e_3 - 2e_1 e_2 e_3 + 2\beta x_3 e_1 e_2 \\
&\quad - 2\beta x_2 e_1 e_3 - 2\beta x_1 e_2 e_3 - e_3, \\
\dot{e}_4 &= -3\Delta\delta e_3 - 2e_4 e_2 e_3 - 3\beta x_3 e_4 e_2 \\
&\quad - 3\beta x_2 e_4 e_3 - 3\beta x_4 e_2 - e_4.
\end{aligned} \tag{7}
$$

So, from the above error dynamics we can conclude that FPS between two identical hyper chaotic system can be achieved.

4. Numerical Simulations

If Perturbation of Parameters of the response system of hyper chaotic Rabinovich-Fabrikant system are zero and $\beta = 0.5$ with the initial conditions of drive system $[x_1(0), x_2(0), x_3(0), x_4(0)] = [0, 2, 0.5, -0.2]$ and response systems $[y_1(0), y_2(0), y_3(0). y_4(0)] = [0.5, 1, -0.1, -0.15]$ respectively.

So, the initial conditions for $[e_1(0), e_2(0), e_3(0), e_4(0)] = [0.5, 0, -0.35, -0.05]$

diagrams of convergence of errors given below are the witness of achieving function projective synchronization between master and slave system.

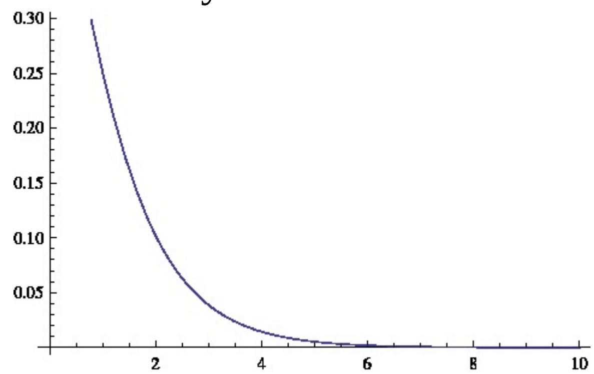

Fig.16 Convergence of error e_1, $t \in [0,10]$

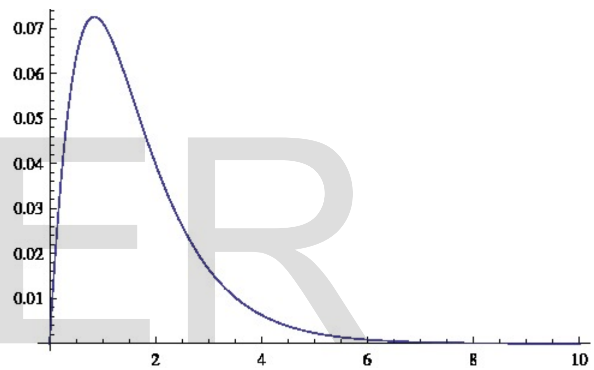

Fig.17 Convergence error of e_2, $t \in [0, 10]$

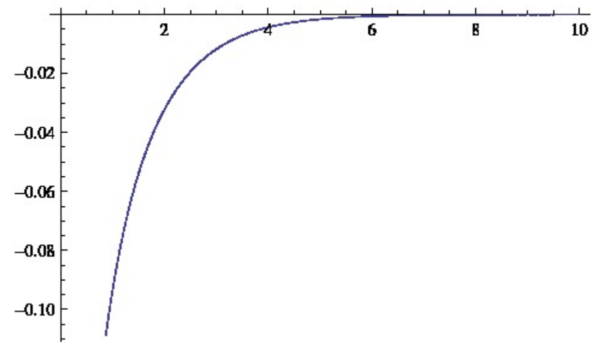

Fig.18 Convergence of error e_3, $t \in [0, 10]$

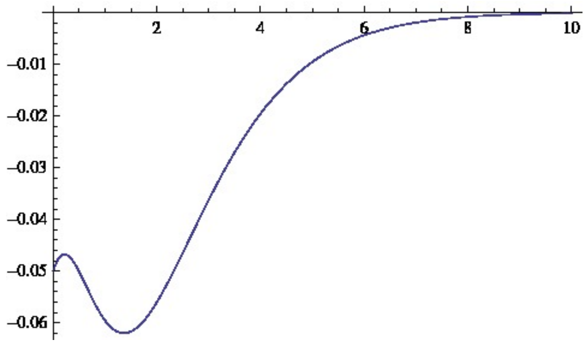

Fig.19 Convergence of error e_4, $t \in [0, 10]$

5.Conclusion:

In this paper, we have investigated function projective synchronization behavior of a new hyper chaotic Rabinovich-Fabrikant system . The results are validated by numerical simulations using mathematica. It has more advantage over other synchronization to enhance security of communicationas function projective synchronization is more unpredictable and moreover it is performed for hyperchaotic system, which makes it more useful.

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